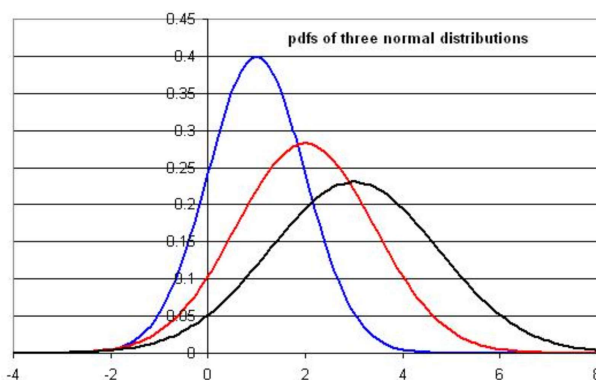




### 3. The Normal Distribution: calculating $\mu$ and/or $\sigma$

1.  $X$  has a normal distribution, and  $P(X > 73.05) = 0.0289$ . Given that the variance of the distribution is 18, find the mean.
2. The random variable  $X \sim N(0, \sigma^2)$ . Given that  $P(-4 < X < 4) = 0.6$ , find the value of  $\sigma$ .
3. The random variable  $X \sim N(2.68, \sigma^2)$ . Given that  $P(X > 2a) = 0.2$  and  $P(X < a) = 0.4$ , find the value of  $\sigma$  and the value of  $a$ .
4.  $X$  is distributed normally,  $P(X \geq 59.1) = 0.0218$  and  $P(X \geq 29.2) = 0.9345$ . Find the mean and standard deviation of the distribution, correct to 3 significant figures.
5. The SigmaMu company makes a soft drink sold in '330 ml' cans. The actual volume of drink in the cans is distributed normally with standard deviation 2.5 ml. To ensure that at least 99% of the cans contain more than 330 ml, find the volume that the company should supply in the cans on average.
6. The life of the Powerhouse battery has a normal distribution with mean 210 hours. It is found that 4% of these batteries operate for more than 222 hours. Find the variance of the distribution, correct to 2 significant figures.
7. The masses of the penguins on an island are found to be normally distributed with mean  $\mu$  and standard deviation  $\sigma$ .
  - (a) Given that 10% of the penguins have a mass less than 18 kg and 5% have a mass greater than 30 kg, calculate the values of  $\mu$  and  $\sigma$ .  
10 penguins are chosen at random
  - (b) Find the probability that at least 4 of them have a mass greater than 25 kg.

8\*. This chart accurately shows the probability density functions for three normal distributions. I estimate that the probability that a variable drawn from the blue distribution (with its peak at 0.4) is negative is 0.25. Can you suggest how I made this estimate? Is it likely to be an over- or under-estimate? Estimate the probabilities that random variables drawn from the other distributions are negative.



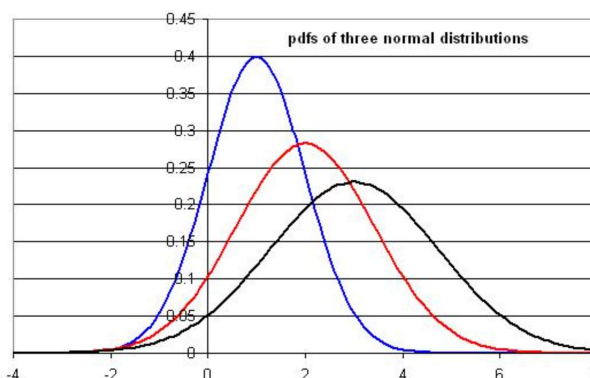
The means and variances of these normal distributions are whole numbers. Can you use your estimates to work out the values for these means and variances, using just your calculator and estimates? Can you specify the values exactly, or can you only specify a certain range of possibilities? (Taken from nrich)



## 7. The Normal Distribution: calculating $\mu$ and/or $\sigma$

1.  $X$  has a normal distribution, and  $P(X > 73.05) = 0.0289$ . Given that the variance of the distribution is 18, find the mean. **65**
2. The random variable  $X \sim N(0, \sigma^2)$ . Given that  $P(-4 < X < 4) = 0.6$ , find the value of  $\sigma$ . **4.75**
3. The random variable  $X \sim N(2.68, \sigma^2)$ . Given that  $P(X > 2a) = 0.2$  and  $P(X < a) = 0.4$ , find the value of  $\sigma$  and the value of  $a$   **$\sigma = 1.99, a = 2.18$**
4.  $X$  is distributed normally,  $P(X \geq 59.1) = 0.0218$  and  $P(X \geq 29.2) = 0.9345$ . Find the mean and standard deviation of the distribution, correct to 3 significant figures. **42.0, 8.47**
5. The SigmaMu company makes a soft drink sold in '330 ml' cans. The actual volume of drink in the cans is distributed normally with standard deviation 2.5 ml. To ensure that at least 99% of the cans contain more than 330 ml, find the volume that the company should supply in the cans on average. **336 ml**
6. The life of the Powerhouse battery has a normal distribution with mean 210 hours. It is found that 4% of these batteries operate for more than 222 hours. Find the variance of the distribution, correct to 2 significant figures. **47**
7. The masses of the penguins on an island are found to be normally distributed with mean  $\mu$  and standard deviation  $\sigma$ .
  - (a) Given that 10% of the penguins have a mass less than 18 kg and 5% have a mass greater than 30 kg, calculate the values of  $\mu$  and  $\sigma$ .  **$\mu = 23.31, \sigma = 4.10$**
  - 10 penguins are chosen at random
  - (b) Find the probability that at least 4 of them have a mass greater than 25 kg. **0.447**

8\*. This chart accurately shows the probability density functions for three normal distributions. I estimate that the probability that a variable drawn from the blue distribution (with its peak at 0.4) is negative is 0.25. Can you suggest how I made this estimate? Is it likely to be an over- or under-estimate? Estimate the probabilities that random variables drawn from the other distributions are negative.



The means and variances of these normal distributions are whole numbers. Can you use your estimates to work out the values for these means and variances, using just your calculator and estimates? Can you specify the values exactly, or can you only specify a certain range of possibilities? (Taken from nrich) **We can estimate the probability of selecting a negative random variable by evaluating the area under the curve in the region of negative  $x$ . Approximating this area using a triangle will give us an over-estimate of the actual probability. Blue curve:  $P(X < 0) \approx 0.5 \times 2 \times 0.25 = 0.25$ , red curve:  $P(X < 0) \approx 0.5 \times 2 \times 0.1 = 0.1$ , black curve:  $P(X < 0) \approx 0.5 \times 2 \times 0.05 = 0.05$ .**

A normal distribution is symmetric about its mean. This allows us to estimate the mean of each distribution by inspection:  $\mu_{\text{blue}} = 1, \mu_{\text{red}} = 2, \mu_{\text{black}} = 3$ . Using this and the above information gives  $\sigma_{\text{blue}}^2 = 1, \sigma_{\text{red}}^2 = 2, \sigma_{\text{black}}^2 = 3$ .