2. Binomial theorem extended: Next level

1. Find the first four terms in the expansion of each of the following in ascending powers of x. State the interval of values of x for which each expansion is valid.
(a) $\sqrt[3]{8-16x}$
(b) $\dfrac{2}{2-x}$
(c) $\dfrac{4x}{\sqrt{4+x^3}}$

2. $f(x) = \dfrac{1+x}{1-2x}$.
(a) Show that the series expansion of $f(x)$ up to and including the x^3 term is $1+3x+6x^2+12x^3$.
(b) State the range of values of x for which the expansion is valid.

3. Expand $\sqrt{1+8x}$ in ascending powers of x up to and including the term in x^3. By giving a suitable value to x, find an approximation for $\sqrt{1.08}$. Deduce approximations for
(a) $\sqrt{108}$
(b) $\sqrt{3}$

4. Given that the coefficient of x^3 in the expansion of $\dfrac{1}{(1+ax)^3}$ -2160,find a.

5. Find the coefficient of x^2 in the expansion of $\dfrac{(1-2x)^2}{(1+x)^2}$.

6. Find the first three terms in the expansion of $\dfrac{\sqrt{1+2x}}{\sqrt{1-4x}}$ in ascending powers of x. State the values of x for which the expansion is valid. By substituting $x = 0.01$ in your expansion, find an approximation for $\sqrt{17}$.

7. Find the first three terms in the expansion in ascending powers of x of $\dfrac{1+2x}{(1-x+2x^2)^3}$.

8. Given that the expansion of $(1+ax)^n$ is $1-2x+\frac{7}{3}x^2+kx^3+\ldots$, find the value of k.

9*. Find the first three terms in the expansion in ascending powers of x of $\dfrac{3+4x+x^2}{\sqrt[3]{1+\frac{1}{2}x}}$. Hence find an approximation to $\int_{-0.5}^{0.5} \dfrac{3+4x+x^2}{\sqrt[3]{1+\frac{1}{2}x}}\mathrm{d}x$

5. Binomial theorem extended: Next level

1. Find the first four terms in the expansion of each of the following in ascending powers of x. State the interval of values of x for which each expansion is valid.
(a) $\sqrt[3]{8-16x}$ $2-\frac{4}{3}x-\frac{8}{9}x^2-\frac{80}{81}x^3, |x|<\frac{1}{2}$
(b) $\dfrac{2}{2-x}$ $1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{8}x^3, |x|<2$
(c) $\dfrac{4x}{\sqrt{4+x^3}}$ $2x-\frac{1}{4}x^4+\frac{3}{64}x^7-\frac{5}{512}x^{10}, |x|<\sqrt[3]{4}$

2. $f(x)=\dfrac{1+x}{1-2x}$.
(a) Show that the series expansion of $f(x)$ up to and including the x^3 term is $1+3x+6x^2+12x^3$.
(b) State the range of values of x for which the expansion is valid. $|x|<\frac{1}{2}$

3. Expand $\sqrt{1+8x}$ in ascending powers of x up to and including the term in x^3. $1+4x-8x^2+32x^3$ By giving a suitable value to x, find an approximation for $\sqrt{1.08}$. 1.039232 Deduce approximations for
(a) $\sqrt{108}$ 10.39232
(b) $\sqrt{3}$ 1.73205

4. Given that the coefficient of x^3 in the expansion of $\dfrac{1}{(1+ax)^3}$ -2160,find a. 6

5. Find the coefficient of x^2 in the expansion of $\dfrac{(1-2x)^2}{(1+x)^2}$. 15

6. Find the first three terms in the expansion of $\dfrac{\sqrt{1+2x}}{\sqrt{1-4x}}$ in ascending powers of x. State the values of x for which the expansion is valid. By substituting $x=0.01$ in your expansion, find an approximation for $\sqrt{17}$. $1+3x+\frac{15}{2}x^2, |x|<\frac{1}{4}, 4.123$

7. Find the first three terms in the expansion in ascending powers of x of $\dfrac{1+2x}{(1-x+2x^2)^3}$. $1+5x+6x^2$

8. Given that the expansion of $(1+ax)^n$ is $1-2x+\frac{7}{3}x^2+kx^3+\ldots$, find the value of k. $-\frac{56}{27}$

9*. Find the first three terms in the expansion in ascending powers of x of $\dfrac{3+4x+x^2}{\sqrt[3]{1+\frac{1}{2}x}}$. Hence find an approximation to $\int_{-0.5}^{0.5}\dfrac{3+4x+x^2}{\sqrt[3]{1+\frac{1}{2}x}}\mathrm{d}x$ $3+\frac{7}{2}x+\frac{1}{2}x^2, 3\frac{1}{24}$