4. Sequences: [The sum of a geometric sequence

1. Find the sum, for the given number of terms, of each of the following geometric series. Give decimal answers correct to 4 decimal places.
(a) $2 + 6 + 18 + \ldots$, 10 terms
(b) $12 - 4 + \frac{4}{3} - \ldots$, 10 terms

2. Find the sum of the following geometric series.
(a) $1 + 2 + 4 + \ldots + 1024$
(b) $1 - 2 + 4 - \ldots + 1024$
(c) $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots + \frac{1}{1024}$
(d) $1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n}$
(e) $81 - 27 + 9 - \ldots + \frac{1}{(-3)^n}$

3. How many terms of the geometric series $1 + 2 + 4 + 8 + \ldots$ must be taken for the sum to exceed 10^{11}

4. A geometric series has first term a and common ratio r. The sum of the first two terms of the series is 4.48. The sum of the first four terms is 5.1968. Find the two possible values of r.

5. The first term of a geometric series is a and the common ratio is $\sqrt{3}$. Show that $S_{10} = 121a(\sqrt{3} + 1)$

6. An explorer sets out across the desert with 100 litres of water. She uses 6 litres on the first day. On subsequent days she rations herself to 95% of the amount she used the day before. Show that she has enough to last for 34 days, but no more. How much will he then have left?

7**. Given that the coefficients of x, x^2 and x^4 in the expansion of $(1 + kx)^n$, where $n \geq 4$ and k is a positive constant, are the consecutive terms of a geometric series,
(a) show that that $k = \frac{6(n-1)}{(n-2)(n-3)}$
(b) Given further that both n and k are positive integers, find all the possible pairs of values for n and k. You should show clearly how you know that you have found all possible pairs of values.

(c) For the case when $k = 1.4$, find the value of the positive integer n. (AEA Specimen Q6)

8**. In this question a and x are positive real numbers, with $a > 1$, $x \neq a$, $x \neq 1$ and n is an integer with $n > 1$. Dr. Ellis was confused about the rules of logarithms and thought that
$\log_a x^n = (\log_a x)^n \quad (1)$
(a) Given that x satisfies statement (1), find x in terms of a and n.
Dr. Ellis also thought that
$\log_a x + \log_a x^2 + \ldots + \log_a x^n = \log_a x + (\log_a x)^2 + \ldots + (\log_a x)^n \quad (2)$
(b) For $n = 3$, x_1 and x_2 ($x_1 > x_2$) are the two values of x that satisfy statement (2).
(i) Find, in terms of a, an expression for x_1 and an expression for x_2.
(ii) Find the exact value of $\left(\log_a \frac{x_1}{x_2}\right)$.
(c) Show that if $\log_a x$ satisfies statement (2) then $2(\log_a x)^n - n(n+1)\log_a x + (n^2 + n - 2) = 0$
(Adapted from AEA 2012 Q5)

Fred and Amy's Maths Shack

9. Sequences: [The sum of a geometric sequence

1. Find the sum, for the given number of terms, of each of the following geometric series. Give decimal answers correct to 4 decimal places.
(a) $2 + 6 + 18 + \ldots$, 10 terms 59048
(b) $12 - 4 + \frac{4}{3} - \ldots$, 10 terms 8.9998

2. Find the sum of the following geometric series.
(a) $1 + 2 + 4 + \ldots + 1024$ 2047
(b) $1 - 2 + 4 - \ldots + 1024$ 683
(c) $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots + \frac{1}{1024}$ $\frac{341}{1024}$
(d) $1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n}$ $2 - (\frac{1}{2})^n$
(e) $81 - 27 + 9 - \ldots + \frac{1}{(-3)^n}$ $\frac{1}{4}(243 + (-\frac{1}{3})^n)$

3. How many terms of the geometric series $1 + 2 + 4 + 8 + \ldots$ must be taken for the sum to exceed 10^{11} 37

4. A geometric series has first term a and common ratio r. The sum of the first two terms of the series is 4.48. The sum of the first four terms is 5.1968. Find the two possible values of r.
$r = \pm 0.4$

5. The first term of a geometric series is a and the common ratio is $\sqrt{3}$. Show that $S_{10} = 121a(\sqrt{3} + 1)$

6. An explorer sets out across the desert with 100 litres of water. She uses 6 litres on the first day. On subsequent days she rations herself to 95% of the amount she used the day before. Show that she has enough to last for 34 days, but no more. How much will he then have left?
0.979 litres

7**. Given that the coefficients of x, x^2 and x^4 in the expansion of $(1 + kx)^n$, where $n \geq 4$ and k is a positive constant, are the consecutive terms of a geometric series,
(a) show that that $k = \frac{6(n-1)}{(n-2)(n-3)}$
(b) Given further that both n and k are positive integers, find all the possible pairs of values for n and k. You should show clearly how you know that you have found all possible pairs of values. $(n, k) = (4, 9)$ or $(5, 4)$. All other values give non-integer k: $(6, 2.5)$, $(7, 1.8)$, $(8, 1.4)$, $(9, \frac{8}{7})$, $(10, \frac{27}{28})$; in the latter k is less than 1. Since the denominator becomes larger at a greater rate than the numerator that all other k will be less than 1.
(c) For the case when $k = 1.4$, find the value of the positive integer n. 8 (AEA Specimen Q6)

8**. In this question a and x are positive real numbers, with $a > 1$, $x \neq a$, $x \neq 1$ and n is an integer with $n > 1$. Dr. Ellis was confused about the rules of logarithms and thought that
$\log_a x^n = (\log_a x)^n \quad (1)$
(a) Given that x satisfies statement (1), find x in terms of a and n. $x = a^{n^{\frac{1}{n-1}}}$
Dr. Ellis also thought that
$\log_a x + \log_a x^2 + \ldots + \log_a x^n = \log_a x + (\log_a x)^2 + \ldots + (\log_a x)^n \quad (2)$
(b) For $n = 3$, x_1 and x_2 $(x_1 > x_2)$ are the two values of x that satisfy statement (2).
(i) Find, in terms of a, an expression for x_1 and an expression for x_2. $x_1 = a^{\frac{-1+\sqrt{21}}{2}}$, $x_2 = a^{\frac{-1-\sqrt{21}}{2}}$
(ii) Find the exact value of $\left(\log_a \frac{x_1}{x_2}\right)$ $\sqrt{21}$.
(c) Show that if $\log_a x$ satisfies statement (2) then $2(\log_a x)^n - n(n+1)\log_a x + (n^2 + n - 2) = 0$
(Adapted from AEA 2012 Q5)